Chapter 1

Easy Problems

1.1. $y = \tan x$

1.2. $f(x) = g(x)\ln(g(x))$.

1.3. $y = \arctan x = \tan^{-1} x$

1.4. $y = \arcsin(x)$

1.5. $y = (x+1)\ln(x+1)$.

Chapter 2

Probability Spaces

2.1. A coin is weighted so that heads is four times as likely as tails. Find the probability that: (a) tails appears, (b) heads appears

2.2. Under which of the following functions does $S = \{a_1, a_2\}$ become a probability space?

(a) $P(a_1) = \frac{1}{3}$, $P(a_2) = \frac{1}{2}$ (b) $P(a_1) = \frac{3}{4}$, $P(a_2) = \frac{1}{4}$
(c) $P(a_1) = 1$, $P(a_2) = 0$ (d) $P(a_1) = \frac{5}{4}$, $P(a_2) = -\frac{1}{4}$

Appendix A

Solutions

1.1

$$\begin{aligned} y &= \tan x \\ &= \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= \frac{\cos x}{\cos x} + \sin x \times \frac{-1}{\cos^2 x} \times - \sin x \\ &= 1 + \tan^2 x \\ &= \sec^2 x. \end{aligned}$$

1.2

$$\begin{aligned} f'(x) &= g'(x) \ln(g(x)) + \frac{g(x)}{g(x)} g'(x) \\ &= g'(x)(1 + \ln(g(x))). \end{aligned}$$

1.3

$$\tan y = x$$

diff w.r.t. x:

$$\begin{aligned} \sec^2 y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\ &= \frac{1}{1 + \tan^2 y} \\ &= \frac{1}{1 + x^2} \end{aligned}$$

1.4

$$\sin(y) = x$$

diff. w.r.t. x:

$$\begin{aligned}
\cos y \frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{1}{\cos y} \\
&= \frac{1}{\sqrt{1 - \sin^2 y}} \\
&= \frac{1}{\sqrt{1 - x^2}}.
\end{aligned}$$

1.5

$$\begin{aligned}
\frac{dy}{dx} &= \ln(x+1) + \frac{x+1}{x+1} \\
&= 1 + \ln(x+1).
\end{aligned}$$

2.1 Let $p = P(T)$, then $P(H) = 4p$. We require $P(H) + P(T) = 1$, so $4p + p = 1$, hence $p = \frac{1}{5}$. Therefore: (a) $P(T) = \frac{1}{5}$, (b) $P(H) = \frac{4}{5}$

2.2 2.2b and 2.2c